# Resources for Quantum Information Processing

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### PROGRAM DESCRIPTION

The aim of this program is to further our knowledge of the resources necessary for quantum information processing, including quantum **measurement**, quantum **control**, and **entanglement**, to apply this knowledge to practical QIP as well as exploring its more **fundamental aspects**.

## 1. Quantum Measurement

Following on from previous work on quantum state tomography for $d$-dimensional systems [Roy and Scott, J. Math. Phys. **48**, 072110 (2007)], we considered ancilla-assisted quantum process tomography. For unital channels (those that preserve the identity) we showed that weighted unitary 2-designs define optimal measurements on the system-ancilla output state [Scott, J. Phys. A **41**, 055308 (2008)]. Examples of such 2-designs include complete sets of mutually unbiased unitary-operator bases, each of which specifies a minimal series of optimal orthogonal measurements.

## 2. Quantum Control

This year we have continued to investigate feedback control of quantum systems for rapid state preparation. Previously we had considered two protocols for rapid-state purification of qubits [see e.g. Griffith, Hill, Ralph, Wiseman, and Jacobs, Phys. Rev. B **75**, 014511 (2007)]. Both schemes use a continuous measurement of z-spin and seek to minimize the time required to purify the conditional state. The first protocol uses only open-loop control, to keep the qubit Bloch vector aligned to the z-axis. The second keeps the Bloch vector orthogonal to the z-axis, which requires closed-loop (feedback) control. This year we analysed these protocols using the Hamilton-Jacobi-Bellman equation and theorems from optimal control theory [Wiseman and Bouten, Quantum Information Processing **7**, 77 (2008)]. We proved that these two protocols are optimal for their respective goals: minimizing the average time to attain a given purity, and maximizing the average purity at a given time. Previously we had also considered continuous measurement of the z-spin of a $d=2j+1$ dimensional system, and developed a control algorithm giving an increase in average purification rate of order d [Combes and Jacobs, Phys. Rev. Lett. **96**, 010504 (2006)]. This year we introduced a new algorithm giving an increase in the rate of purification scaling as $d^2$, and also introduced an analogous algorithm for a register of $n$ qubits, and proved an increase in the purification rate by a factor of order $n$ [Combes, Wiseman and Jacobs, Phys. Rev. Lett. **100**, 160503 (2008)]. Moreover, unlike our earlier algorithms, these new algorithms can also be used for rapid read-out.

In the area of open-loop control, we considered all-optical control of quantum dot spins via auxiliary exciton (electron-hole) states, which offers the promise of fast gates. For simplicity we first considered a single-qubit phase gate, and compared adiabatic pulse sequence to a conventional (simple dynamic) implementation [Gauger, Benjamin, Nazir, and Lovett, Phys. Rev. B **77**, 115322 (2008)]. We found that the adiabatic gate can be extremely robust against the combined effect of all principal sources of decoherence, with an achievable fidelity of 0.999 even at finite temperature. Moreover this performance can be obtained with an increase in gate-time to merely one order of magnitude times the conventional approach. We then considered entangling operations between two quantum dot spins, and found similarly that the adiabatic approach was much more robust [Gauger, Nazir, Benjamin, Stace, and Lovett, New J. Phys. **10**, 073016 (2008)].

## 3. Entanglement

Quantum reference systems (such as phase references) are a resource for quantum information processing intimately related to entanglement. As previously reported, we found a triality between the system's ability to do local mechanical work, its ability to do "logical work" due to its accessible entanglement, and its ability to act as a shared reference system [Vaccaro, Anselmi, Wiseman, and Jacobs, Phys. Rev. A **77**, 032114 (12 pages) (2008)]. Following on from that, we have investigated the optimal reference states for maximizing accessible entanglement under the local particle number superselection rule. For a system consisting of a single shared particle, we find analytically the family of optimal phase reference states having fixed particle number [White, J.A. Vaccaro, and H.M. Wiseman, Phys. Rev. A to be published (2009)]. These reference states are similar, but not identical, to states with a minimum uncertainty in relative-phase.

Another area with intimate connections to entanglement is the question of local distinguishability among multipartite states drawn from some ensemble. We obtained some interesting result relating to completely entangled subspaces (a subspace of a multipartite Hilbert space which contains no product states). Such subspaces can be large, with a known maximum size, $s_{\max}$, approaching the full dimension of the system, $D$. We have shown that almost all subspaces with dimension $s \leq s_{\max}$ are completely entangled [Walgate and Scott, J. Phys A **41** 375305 (2008)]. Using this result, we showed that for almost all ensembles of $n$ pure states, the states are distinguishable by local measurements of the parties involved if and only if $n \leq D - s_{\max}$. This condition holds for both separable and nonseparable pure states; surprisingly, entanglement makes no difference.

Finally, we studied the probabilistic preparation of entangled states of an ensemble of $N$ two-level atoms in a cavity QED situation where $N$ is known exactly. The atoms are effective two-level atoms driven with laser fields and coupled to the cavity field, and the photon emissions from the cavity are monitored by an (assumed perfect) detector. Conditioned on the detector clicks, "entangled-state cycles" arise when the (Raman) transition rates between the two atomic levels are set equal. During each cycle the $N$-qubit state switches, with each cavity photon emission, between the states $|N/2,m\rangle \pm |N/2,-m\rangle$, where $|N/2,m\rangle$ is a Dicke state in a rotated collective basis [Chia and Parkins, Physical Review A **77**, 033810, (2008)]. The quantum number $m>0$ which distinguishes the particular cycle, is determined by the photon counting record and varies randomly from one trajectory to the next. For even $N$ it is also possible, under the same conditions, to prepare probabilistically (but in steady state) the Dicke state $|N/2,0\rangle$, i.e., an $N$-qubit state with $N/2$ excitations, which is of particular interest in the context of multipartite entanglement.

## 4. Fundamental aspects of QIP

Quantum teleportation is a primitive in quantum information processing, and has continuous-variable as well as qubit versions. In the CV case perfect teleportation requires an infinite resource – an infinitely squeezed state, with $e^{-r}\to 0$. For finite squeezing the fidelity of the output state will be less than unity. We have shown a connection between this fidelity and the sub-Planck structure in the Wigner function of the system to be teleported [Scott and Caves, Annals of Physics. **323**, 2685 (2008)]. Specifically, for large squeezing ($e^{-r}$ small), the shortfall in the output fidelity is proportional to $e^{-r}$ times the integral of the modulus squared of the gradient of the Wigner function. This reflects the fact that high fidelity in the output state requires a squeezing large enough that the smallest sub-Planck structures in an input pure state are teleported faithfully. Alternatively, it can be understood from the fact that the smallness of scale of the fine-scale structure implies a largeness of scale of the extent of the Wigner function. We treated numerous specific examples, including coherent, number, and random states and states produced by chaotic dynamics, and also a generalization to mixed states.

On the more speculative side, in an invited article [Wiseman and Eisert, “Nontrivial quantum effects in biology: A skeptical physicists’ view”, in *Quantum Aspects of Life*, (Imperial College Press, London, 2008) edited by D. Abbot, P. C.W. Davies, and A.K. Pati], we have critiqued various claims that quantum information processing occurs in biological systems, but also pointed out some in-principle differences in predictability between classical computers and brains. We have also continued our analysis of quantum information processing from the perspective of retrocausal quantum mechanics [Pegg, Found. Physics, **38**, 648-658, 2008; Stud. Hist. Phil. Mod. Phys. **39**, 830–840, 2008].

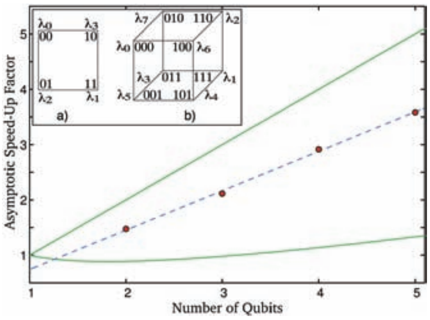

**FIGURE 1**

The asymptotic (in time) speed-up in the mean time for a quantum register to reach a given level of purity, as a function of the number of qubits in the register. The dashed line is a linear fit. The solid lines are exact analytical bounds, both of which are linear in $n$ for $n$ large. The feedback algorithm requires rapid logical operations to re-arrange logical eigenstates; the inset shows the optimal eigenvalue arrangement for a) a two qubit register, and b) a three qubit register.

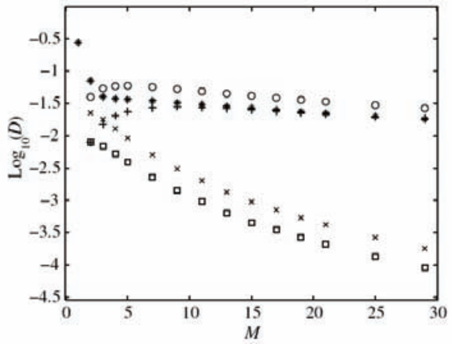

**FIGURE 3**

A comparison of the relative difference $D$ in accessible entanglement of a shared single particle using different reference states of fixed total particle number $M$, in comparison to the optimal such reference state. The comparison states are: the shared trucated phase states (o); trucanted two-mode versions of the coherent states (∗); binomial states (+); Summy-Pegg phase optimised states (x), and Berry-Wiseman phase optimized states (□).The effectiveness of all states increases as $M$ increases.

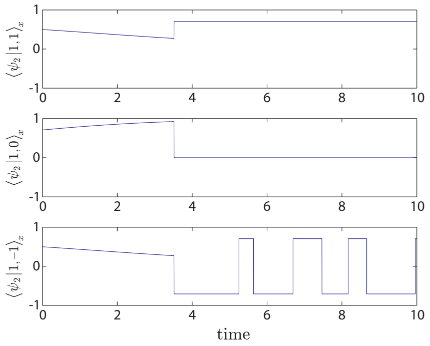

**FIGURE 4**

Example trajectory for the simplest case $N=2$, with an initial state $|00\rangle$ (both atoms spin down in the z-direction). In this trajectory the system settles into an entangled-state cycle after the first jump, with the two entangled states being $|01\rangle+|10\rangle$ and $|00\rangle+|11\rangle$. In other trajectories, the system goes to the steady-state $|01\rangle-|10\rangle$.

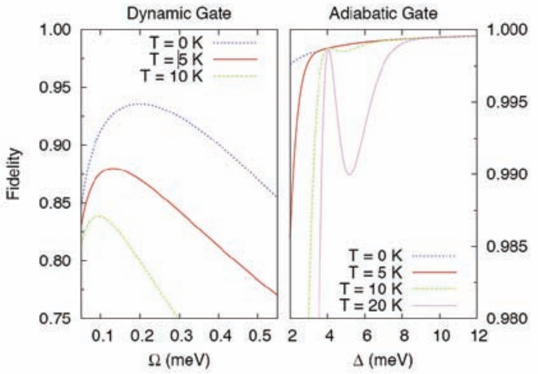

**FIGURE 2**

Comparison of the conventional versus the adiabatic gate for a $\pi$-phase gate operation by showing the overall gate fidelity. Left: fidelity of the conventional operation as a function of the coupling strength. Right: fidelity of the adiabatic operation, for a fixed coupling strength of 1 meV, as a function of the detuning.

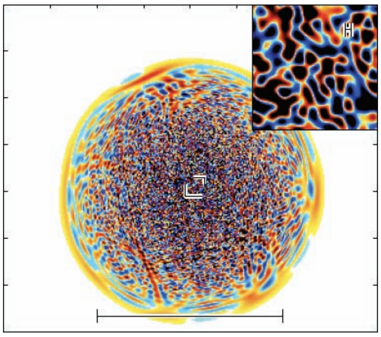

**FIGURE 5**

Wigner function for a randomly generated pure state with support over 100 harmonic oscillator eigenstates. The inset shows the fine-scale (sub-Planck structure) which requires a highly squeezed entangled resource to be faithfully teleported, and goes hand-in-hand with the large extent of the Wigner function in phase space (the large scale bar has a length of 20 harmonic oscillator units).