Approximate method for treating dispersion in one-way quantum channels

T. M. Stace[1] and H. M. Wiseman[2]
[1]*DAMTP, University of Cambridge, Cambridge CB30WA, United Kingdom*
[2]*Centre for Quantum Computer Technology, Center for Quantum Dynamics, School of Science, Griffith University, Nathan 4111, Australia*
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Coupling the output of a source quantum system into a target quantum system is easily treated by cascaded systems theory if the intervening quantum channel is dispersionless. However, dispersion may be important in some transfer protocols, especially in solid-state systems. In this paper we show how to generalize cascaded systems theory to treat such dispersion, provided it is not too strong. We show that the technique also works for fermionic systems with a low flux, and can be extended to treat fermionic systems with large flux. To test our theory, we calculate the effect of dispersion on the fidelity of a simple protocol of quantum state transfer. We find good agreement with an approximate analytical theory that had been previously developed for this example.

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I. INTRODUCTION

Theoretical methods for treating nonideal components in quantum networks is an important task for quantifying imperfections in experiments. One common example is photon loss in optical channels, which can be treated by invoking a fictitious beam splitter that mixes the channel mode with other experimentally inaccessible modes [1]. The component of the channel mode reflected by the beam splitter therefore corresponds to photon loss. This approach also allows inefficient detection to be accurately modeled. In this paper, we present a technique for treating dispersion in the quantum channels connecting cascaded quantum systems.

Cascaded quantum systems are ones for which the output of one system acts as the input for a second system, with no back-action from the second on to the first. Examples include driving atomic systems with nonclassical light [1], and some with an application to quantum information [2]. Methods for simulating these systems have been developed assuming that the intervening channel is dispersionless.

In the absence of dispersion, the input to a downstream system is just a time-delayed version of the upstream output operator. As long as we are only concerned with correlation functions of each subsystem independently, then the time delay is irrelevant, and the system can be described by an ordinary differential equation (a Langevin equation for the system operators, or a master equation for the system state). By contrast, for a dispersive channel, the input to the second system is the output of the upstream system convolved with the channel transfer function. In this case there is no Markovian evolution equation for the system operators or system state. Although the non-Markovian evolution may be soluble in special cases, in general, an exact treatment is impossible.

Dispersion arises when modes acquire a phase after propagation that depends nonlinearly on frequency. Typically, efforts are made to operate optical fibers at the zero-dispersion point in order for this effect to be small, and heterogeneous structures may be used to provide an effectively dispersion free channel. Nevertheless, in some circumstances, it may be desirable to operate in a regime where dispersion is not negligible. Proposals for implementing mesoscopic analogs of optical schemes, such as interferometers [3,4], and quantum state transfer protocol [5] using the quantum Hall effect will necessarily have some dispersion, due to the nonzero mass of quasielectrons in the edge state. In that case, an *ad hoc* approach was used to estimate the effect of dispersion. Another quantum system in which dispersion during propagation is expected to be important is atom lasers [6].

In the example of treating photon loss, an additional element, the beam splitter, is added to an otherwise ideal channel to provide a tractable model. In analogy with this approach, we propose a way to treat dispersion by introducing an extra element to an otherwise ideal (i.e., dispersionless) channel: a resonant, damped cavity, operating in reflection. Near resonance, incident modes suffer a frequency dependent phase shift on reflection depending nonlinearly on their detuning from the resonance. This is broadly the same condition that arises in a dispersing channel, so the goal is to fix the resonance and damping of the cavity to match dispersion as closely as possible.

In essence, the effects associated with non-Markovian dynamics in the channel are accounted for by the (few) extra degrees of freedom introduced by the fictitious cavity. This approach is analogous to stochastic methods for simulating non-Markovian systems by transforming the problem to an equivalent one with Markovian dynamics on a doubled Hilbert space [7].

Since there are only two parameters for the cavity, it is plainly not possible to treat arbitrary dispersion with this approach. However, we show that in simple networks (without feedback or interference between different paths) it is possible to match up to the third order in the dispersion relations. Thus our approach handles channels that are not too dispersive, over the range of input frequencies.

We begin by summarizing the effects of both dispersion and reflection from a cavity. We then derive the conditions for which cavity reflection is a good approximation to a dispersive channel, relating the frequency and damping of the fictitious cavity to the physical parameters describing the dis-

persive channel. We then make some brief comments on the restrictions of this approach to channels in feedback systems and fermionic systems, and derive a master equation for describing the dynamics for subsystems connected by a one-way quantum channel. The paper concludes with a simple example illustrating the application of the approach to treating quantum state transfer over weakly dispersive channels.

II. PRELIMINARIES

Consider the case of a noninteracting quantum field propagating in one dimension. Let $\hbar=1$. Then at the origin (e.g., point of emission) the field can be expanded in terms of eigenmode operators

$$\psi(0)=\sum_{\omega} b_{\omega}e^{-i\omega t}. \tag{1}$$

Here we are implicitly considering only modes propagating in the positive direction. This limitation will be justified by later (more restrictive) assumptions. The use of a discrete sum is for notational convenience only.

In general, the state of the field at some position L, is given by

$$\psi(L)=\sum_{\omega} b_{\omega}e^{-i\omega t}H_L(\omega), \tag{2}$$

where $H_L(\omega)$ is the transfer function of the channel. For a passive lossless channel, the transfer function is given by $H_L(\omega)=e^{ik(\omega)L}$, where $k(\omega)$ is the dispersion relation for the channel. A widely applicable expression for the dispersion relation for dispersing channels is $\omega=vk+\alpha k^2$, so we take

$$k(\omega)=(2\alpha)^{-1}(-v+\sqrt{v^2+4\alpha\omega}). \tag{3}$$

The group velocity is

$$u=\left.\frac{\partial\omega}{\partial k}\right|_{\omega=\bar{\omega}}=\sqrt{v^2+4\alpha\bar{\omega}}, \tag{4}$$

where $\bar{\omega}$ is the carrier frequency. For a free nonrelativistic particle $v=0$ and $\alpha=(2m)^{-1}$. For an electron propagating in an edge state typically $\alpha\bar{\omega}\ll v^2$ so that $u\approx v$ [5]. (We will return later to the problem that an electron is not a boson.)

Now compare the above expressions to a dispersionless boson field. At the origin we again have

$$\phi(0)=\sum_{\omega} b_{\omega}e^{-i\omega t}. \tag{5}$$

The (non)-dispersion relation is $\omega=ck$, so at position l the field is

$$\phi(l)=\sum_{\omega} b_{\omega}e^{-i\omega t+i\omega l/c}. \tag{6}$$

If, however, we also include (a) a global phase shift and (b) bouncing off a single-mode cavity of central frequency ω_{f} and linewidth γ_{f} then

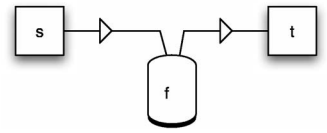

FIG. 1. Schematic of a triply cascaded system. The output of subsystem s reflects off subsystem f, and the reflected field drives subsystem t. No signal propagates in reverse.

$$\phi(l)=\sum_{\omega} b_{\omega}e^{-i\omega t+i\omega l/c+i\theta}\frac{\gamma_{\mathsf{f}}+2i(\omega-\omega_{\mathsf{f}})}{\gamma_{\mathsf{f}}-2i(\omega-\omega_{\mathsf{f}})}. \tag{7}$$

For this result, see, for example, Ref. [8]. This is valid only if the Markovian description of the coupling of the external field to a single mode can be used, which requires

$$\Delta_{\mathsf{f}},\gamma_{\mathsf{f}},\delta\omega\ll\bar{\omega}, \tag{8}$$

where $\Delta_{\mathsf{f}}=\bar{\omega}-\omega_{\mathsf{f}}$ and $\delta\omega$ is the uncertainty in the energy.

III. FEEDFORWARD

Consider the case where the output of system s (source) is the input to system t (target). To model dispersion in the propagation between s and t we consider a nondispersing reflecting off an intermediate (fictitious) cavity mode c_{f}, as shown in Fig. 1. From Eqs. (2) and (7), this will work if we can make the approximation

$$\frac{(-v+\sqrt{v^2+4\alpha\omega})L}{2\alpha}\approx\frac{\omega l}{c}+\theta+2\arctan\frac{2(\omega-\omega_{\mathsf{f}})}{\gamma_{\mathsf{f}}}. \tag{9}$$

In this feedforward case the time delay l/c in the propagation, and the absolute phase of the field θ, are irrelevant to how system t responds to the output of system s, as long as any classial driving fields have their timings and phases adjusted appropriately. Thus we can always choose l and θ so that the constant and linear term in the expansion of the left-hand side of Eq. (9) around $\bar{\omega}$ agrees with the right-hand side. Thus in choosing γ_{f} and ω_{f} we need consider only higher order derivatives. Since we have two free parameters it is natural to look at the second and third derivatives. Equating second and third derivative gives

$$\alpha L/u^3=16\gamma_{\mathsf{f}}\Delta_{\mathsf{f}}/(\gamma_{\mathsf{f}}^2+4\Delta_{\mathsf{f}}^2)^2, \tag{10}$$

$$6\alpha^2L/u^5=16\gamma_{\mathsf{f}}(12\Delta_{\mathsf{f}}^2-\gamma_{\mathsf{f}}^2)/(\gamma_{\mathsf{f}}^2+4\Delta_{\mathsf{f}}^2)^3. \tag{11}$$

Solving for Δ_{f} and γ_{f} yields

$$\gamma_{\mathsf{f}}^2=12\Delta_{\mathsf{f}}^2[1+O(\sqrt{\alpha/Lu})], \tag{12}$$

$$\Delta_{\mathsf{f}}^2=\frac{\sqrt{3}u^3}{8L\alpha}[1+O(\sqrt{\alpha/Lu})]. \tag{13}$$

The error is small when $\alpha\ll Lu$. This is equivalent to $\tau_p\ll\tau_d$, where $\tau_p=L/u$ is the propagation time, and $\tau_d=L^2/\alpha$ is the time for a pulse to disperse over a length scale $\sim L$.

In the weak dispersion limit of $v^2\gg\alpha\bar{\omega}$, we have $\Delta_{\mathsf{f}}^2/\bar{\omega}^2=O(v^3/\alpha\bar{\omega}^2L)=O(v/\bar{\omega}L)O(v^2/\alpha\bar{\omega})\gg O(v/\bar{\omega}L)=O(1/\bar{k}L)$. Thus from Eq. (8) we have

$$\bar{k}L \gg 1. \tag{14}$$

In the opposite limit of $v^2 \ll \alpha\bar{\omega}$, we have $\Delta_f^2/\bar{\omega}^2 = O(\sqrt{\alpha/\bar{\omega}}/L) = O(1/\bar{k}L)$. Thus Eq. (14) applies in all regimes. It might seem surprising that our description puts a *lower* limit on the propagation distance, that it should be much longer than a mean wavelength. This can be understood as follows. If the dispersion were significant [such that it is necessary to match up to the third derivative in Eq. (9)] over the distance of a wavelength, the problem would be so non-Markovian that the cavity description would necessarily fail. If it is deemed necessary only to match up to the second derivative then, in principle, Eq. (14) need not hold. However, on physical grounds the second system cannot be within a wavelength or so of the first without a breakdown of cascaded systems theory altogether. Another consideration on the limitation of validity of the theory is that for the third order expansion to be a good approximation we must have

$$\delta\omega \lesssim \gamma_f. \tag{15}$$

This puts an *upper* bound of L which scales as $(\delta\omega)^{-2}$.

If all of the above conditions hold then we can write down a master equation for the cascaded systems s, c, and t that will be a good description of dispersive propagation from s to t.

IV. FEEDBACK OR INTERFERENCE

In other situations the absolute time delay *does* matter, in particular, with feedback. That is, if s feeds into t which feeds back into s. In that case if we wish to use the master equation description we cannot include a time delay l/c. Thus the *first* derivative term must come from the cavity. This gives

$$(v^2 + 4\alpha\bar{\omega})^{-1/2} L = \frac{4\gamma_f}{(\gamma_f^2 + 4\Delta_f^2)}. \tag{16}$$

Substituting this into Eq. (10) gives

$$\alpha(\gamma_f^2 + 4\Delta_f^2) = 2\Delta_f(v^2 + 4\alpha\bar{\omega}). \tag{17}$$

From Eq. (8) we see that we have an inconsistency. Thus we cannot describe feedback for a dispersive field using this model. On the other hand, if $\alpha=0$ (no dispersion) then we can validly satisfy these equations with $\Delta_f=0$ and $\gamma_f=4v/L$. Interestingly, Eq. (8) again gives Eq. (14).

Another situation where time delays matter, at least the difference between two time delays, is when there are two paths by which system s may affect system t. In that case, if the time difference is comparable to the total propagation time then the same inconsistency as noted above will arise. Thus the applicability of this approach to modeling dispersion is most promising for a simple forward chain, and we concentrate on this for the remainder of this paper.

V. MASTER EQUATION

To begin the quantitative analysis, we derive a general master equation for a triply cascaded system, shown in Fig. 1, where the outer systems are arbitrary, but the subsystem f plays the role of the fictitious cavity introduced to simulate dispersion. We assume that subsystem $i \in \{s, f, t\}$ is linearly coupled to the external modes, b_ω, according to

$$H_{i\text{-coup}} = \sum_\omega \kappa_{i\omega} c_i b_\omega^\dagger + \kappa_{i\omega}^* b_\omega c_i^\dagger. \tag{18}$$

We compute the Heisenberg equation of motion for an arbitrary operator, o_i, of subsystem i, and make the Born-Markov approximation, in which we assume $\kappa_{i\omega} = \sqrt{\gamma_i/2\pi}$ is independent of ω. The resulting equation is a Stratonovich stochastic differential equation. In order to derive a master equation, we convert this into an Itô equation, taking care of the spatial ordering of the three cavities (see, for example, Ref. [9]). Alternatively, we can directly apply the cascaded systems theory of Refs. [10,11], iterating the result to include the third system. The master equation for the state matrix for the triply cascaded quantum system is

$$\dot{\rho} = -i[H_{sys} + \tilde{H}, \rho] + \mathcal{D}[\sqrt{\gamma_s} c_s + \sqrt{\gamma_f} c_f + \sqrt{\gamma_t} c_t]\rho, \tag{19}$$

where

$$\tilde{H} = \frac{i}{2}(\sqrt{\gamma_s \gamma_f} c_s^\dagger c_f + \sqrt{\gamma_f \gamma_t} c_f^\dagger c_t + \sqrt{\gamma_t \gamma_s} c_s^\dagger c_t - \text{H.c.})$$

and we have introduced the Lindblad superoperator $\mathcal{D}[a]\rho = a\rho a^\dagger - (a^\dagger a\rho + \rho a^\dagger a)/2$. This master equation satisfies the requirement that the dynamics in subsystem s is unaffected by the dynamics of subsystems f or t, and subsystem f is unaffected by subsystem t, as implied by the cascaded description. We have also defined the bare Hamiltonian for the uncoupled systems $H_{sys} = H_s + H_f + H_t$. H_s and H_t can be arbitrary, depending on the particular application in mind. The middle subsystem is the fictitious cavity that serves to model dispersion, so we take $H_f = \omega_f c_f^\dagger c_f$.

VI. FERMIONS

The technique described above was formulated for bosons. Where it breaks down for fermions is that the Pauli exclusion principle permits only a single particle per cavity mode, so that the simple linear transformation resulting from reflection off a single cavity mode (7) does not hold. However, if there is at most one fermion involved in the problem, then particle statistics are irrelevant and our approach can be applied. Even if there are many fermions, if the flux is low enough then our approach is applicable. Specifically, for a fermion flux of n per second, the average occupation of the fictitious cavity is at most $N = n/\gamma_f$, so the proposal is restricted to fluxes $n \ll \gamma_f$. That is, $n \ll \sqrt{u^3/L\alpha}$.

One method to extend the regime of validity of our method in fermionic systems is shown in Fig. 2. Here the output from s, plus $M-1$ modes in the vacuum state, are directed through an M-port beam splitter (M-BS) onto M fictitious cavities. In this case, the average number of fermions, N, is distributed over M cavities, so the mean occupation per cavity is N/M, which can be made small for sufficiently large M. The splitting is then reversed, and the M modes drive the final subsystem t. Physically, it is easiest to

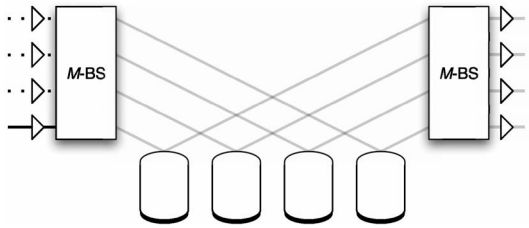

FIG. 2. Fermionic dispersion treated using M-port beam splitters to direct modes onto separate cavities which are subsequently recombined. Dotted lines represent unoccupied modes, and grey lines indicate weakly occupied modes.

imagine that the output of s is a radially symmetric mode, and that the additional $M-1$ vacuum modes are higher-order transverse modes. The fictitious M-BS then could simply be a device that separates M transverse segments (e.g., wedges of a circular wire) and sends them to M fictitious cavities.

The procedure just described leads to the following master equation:

$$\dot{\rho} = -i[H_{\rm sys} + \tilde{H}, \rho] + \frac{1}{M}\sum_{k=1}^{M} \mathcal{D}[\sqrt{\gamma_{\rm s}}c_{\rm s} + \sqrt{\gamma_{\rm f}}c_k + \sqrt{\gamma_{\rm t}}c_{\rm t}]\rho, \tag{20}$$

where

$$\tilde{H} = \frac{i}{2M}\sum_{k=1}^{M} (\sqrt{\gamma_{\rm s}\gamma_{\rm f}}c_{\rm s}^{\dagger}c_k + \sqrt{\gamma_{\rm f}\gamma_{\rm t}}c_k^{\dagger}c_{\rm t} + \sqrt{\gamma_{\rm t}\gamma_{\rm s}}c_{\rm s}^{\dagger}c_{\rm t} - {\rm H.c.}).$$

Here $H_{\rm sys}$ is as before, but with $H_{\rm f}=\Sigma_{k=1}^{M}\omega_{\rm f}c_k^{\dagger}c_k$. It might be thought that a simulation with so many systems would be computationally expensive, but since it is only valid if each fictitious cavity has at most one excitation anyway, the Hilbert space dimension of the fictitious system as a whole is only 2^M. Moreover, the probability that many [that is, $O(M)$] of the cavities are occupied at any one time is very small (since the occupation probability N/M for any one cavity is assumed small). Thus, it should be possible to reduce the number of basis states required for a simulation dramatically.

VII. EXAMPLE: QUANTUM STATE TRANSFER

In order to demonstrate our method, we apply it to a proposed scheme for quantum state transfer [2] between two remote atoms each in a separate cavity, which are connected by an optical channel. This scheme has been adapted to mesoscopic systems, using quantum dots instead of atoms and cavities, and quantum Hall edge states as a communication channel [5], so is relevant to both atom-optical and solid-state systems. This system was sufficiently simple that it was possible to find an approximate analytical expression for the effect of dispersion [5]. We compare this analytic approximation with the more sophisticated method developed above.

The protocol works by controlling the coupling strength between the atom and the cavity, $\Omega_{\rm s,t}(t)$, at each site in such a way that the evolution coherently maps excitation in one atom to excitation in the other atom. For an ideal channel,

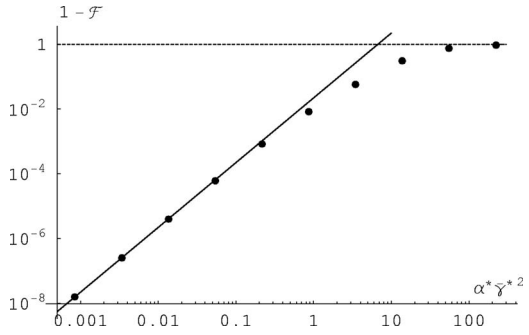

FIG. 3. The infidelity, $1-\mathcal{F}$ versus nondimensional diffusion parameter $\alpha^*\bar{\gamma}^{*2}$. Points are from numerical calculation using a cavity to simulate a dispersive medium. The solid line is the analytic result, taken from [5]. When the dispersion becomes dominant, the infidelity (i.e., error) asymptotes to unity.

one class of suitable control pulses satisfies the relation $\Omega_{\rm s}(t)=\Omega_{\rm t}(\tau_p-t)=\Omega(t)$. Dispersion in the intervening channel has two effects on the fidelity of the transfer protocol. First, the dispersion will broaden the wave packet in the channel so that it will have some reduced fidelity with respect to a comparable wave packet in an ideal, dispersionless channel. Secondly, dispersion modifies the group velocity slightly, so that the wave packet arrives at the destination at a slightly different time. This can be accounted for simply by adjusting the timing and phase of the control fields so that the linear term in $\omega-\bar{\omega}$ in the expansion of Eq. (9), is zero, i.e., $\tau_p=l/c+4\gamma/(\gamma^2+4\Delta^2)\approx l/c+\sqrt{3}/2\Delta$. For the purposes of feedforward simulation, we can take $\tau_p=l/c=0$, so the conditions on the driving fields for optimal transfer is $\Omega_{\rm s}(t)=\Omega_{\rm t}(\sqrt{3}/2\Delta-t)$.

For this model we consider $H_{i=\rm s,t}=\omega_i(c_i^{\dagger}c_i+a_i^{\dagger}a_i)+\Omega_i(t)\times(c_i^{\dagger}a_i+a_i^{\dagger}c_i)$, where c_i are cavity mode annihilation operators, and a_i are atomic lowering operators for each subsystem i, and $\Omega_i(t)$ is a controllable coupling between the atom and cavity mode. We assume the ideal case, $\omega_i=\bar{\omega}$ and $\gamma_{\rm s,t}=\bar{\gamma}$. Moving to the usual interaction frame, the system Hamiltonian is

$$H_{\rm sys} = \sum_{i=\rm s,t}\Omega_i(t)(c_i^{\dagger}a_i + a_i^{\dagger}c_i) - \Delta c_{\rm f}^{\dagger}c_{\rm f}. \tag{21}$$

We assume the system starts in the state $|e,0;0;g,0\rangle$, where $|{\rm atom_s},{\rm cavity_s};{\rm cavity_f};{\rm atom_t},{\rm cavity_t}\rangle$ denotes the states of the three subsystems expressed in the energy eigenbasis of the atoms and cavities. Because there is at most one excitation, particle statistics are irrelevant [5], and there is no need for more than one fictitious cavity.

We can now solve Eq. (19) for the state matrix of the system, which is spanned by the states

$$\{|g,0;0;g,0\rangle,\quad |e,0;0;g,0\rangle,\quad |g,1;0;g,0\rangle,\quad |g,0;1;g,0\rangle,$$

$$|g,0;0;g,1\rangle,\quad |g,0;0;e,0\rangle\}.$$

We use a simple pulse sequence that implements state transfer $\Omega_{\rm s,t}(t)=\bar{\gamma}\,{\rm sech}(\bar{\gamma}t/2)/2$ [12]. Recall that we are using the standard convention for cascaded systems in that the origin

of time for system t is delayed with respect to that for system s.

Recall that the conditions for the cavity to accurately simulate weak dispersion are $\Delta^2=\sqrt{3}u^3/8\alpha L$ and $\gamma^2=12\Delta^2$, so we solve the master equation, Eq. (19), using these parameters. In order to analyze the dependence of the infidelity, given by $\bar{\mathcal{F}}=1-\mathcal{F}$, where $\mathcal{F}$ is the fidelity of the transfer, as a function dispersion, we nondimensionalize the parameters, thus $\alpha^*=\alpha/Lu$, $\Delta^*=\Delta L/u$, and $\gamma^*=\gamma L/u$. In Fig. 3 we compare the results of numerical simulations with the heuristic analytic expression given in [5]. In that work it was found that the infidelity due to dispersion is given by

$$\bar{\mathcal{F}}=(\alpha^*\bar{\gamma}^{*2})^2/45, \tag{22}$$

in the weakly dispersive limit, $\alpha^*\bar{\gamma}^{*2}\ll 1$. In this regime, both approaches are valid and there is very good agreement, lending credibility to both. But our method shows significant deviation from the approximate result even for $\alpha^*\bar{\gamma}^{*2}\gtrsim 1$, for which $\bar{\mathcal{F}}$ is still small (of order 10^{-2}). This regime is at the limit of validity of our approach according to Eq. (15) if we say $\delta\omega\sim\gamma$.

VIII. CONCLUSION

In this paper we have presented a numerical method for modeling the effect of dispersion in quantum channels connecting a source system to a target system. The method is approximate and can treat dispersion that is not too strong. We have also shown how to extend the approach to treat fermionic systems with large flux. Applying our method to a simple example for which there existed a previous *ad hoc* analytical result we showed good agreement between the two methods. For more complicated scenarios, analytical approaches are unlikely to be possible and our technique may be a practical approach.

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