

Quantum memory scheme based on optical fibers and cavities

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An optical quantum memory scheme using two narrow-linewidth cavities and some optical fibers is proposed. The cavities are connected via an optical fiber, and the gap of each cavity can be adjusted to allow photons with a certain bandwidth to transmit through or reflect back. Hence, each cavity acts as a shutter and the photons can be stored in the optical fiber between the cavities at will. We investigate the feasibility of using this device in storing a single photon. We estimate that with current technology storage of a photon qubit for up to 50 clock cycles (round trips) could be achieved with a probability of success of 85%. We discuss how this figure could be improved.

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I. INTRODUCTION

Quantum memory is a key component in many optical quantum information protocols [1], from quantum repeaters [2] to quantum computation [1]. Various quantum memory schemes for storing photonic quantum states have been proposed, such as the use of a single atom in a high- Q cavity [3], all-optical approaches [4], and storing photons in atomic ensembles with the electromagnetically induced transparency (EIT) technique [5–8] and without the EIT technique [9]. Atomic memory schemes have attracted much interest in recent years. Atomic memory has been used to shape photonic pulses [10], to generate correlated photon pairs [11], to produce entanglement between two atomic ensembles [12], and to perform quantum communication [13,14]. However, all these schemes have difficulties. It is hard to efficiently couple a photon with an atom in a high- Q cavity, all-optical approaches have significant photon loss at switches, and atomic memory schemes have a limit in absorbing photons in atoms [15], can suffer from poor quantum state fidelity [9], and require narrow-linewidth photons [16]. Given these difficulties it is of interest to explore alternatives. Here we analyze an all-optical scheme, and estimate its performance with current technology and how it might be improved.

The paper is set out in the following way. In Sec. II we discuss the principles of our scheme, followed by a derivation of the probability of success in Sec. III. In Sec. IV we present results on the probability of success against mirror reflectance and fiber attenuation to quantify the feasibility of this scheme. In Sec. V we discuss what we have found and suggest ways to circumvent problems to improve our scheme. We conclude in Sec. VI.

II. PRINCIPLE OF OUR SCHEME

In this paper, we investigate a quantum memory scheme consisting of two optical cavities connected via an optical fiber, as depicted in Fig. 1. We consider cavity mirrors with reflectance R , transmittance T , and absorptance A . Each cavity allows narrow-wavelength photons to transmit through

with variable probability depending on the mirror separation (see Fig. 2). The scheme works as follows: Imagine a photon is traveling from left to right in an optical fiber and it has come to an optical cavity as shown in Fig. 1. Initially, the left cavity is adjusted to have its mirrors separated at a distance d_{high} such that the photon has a high probability of transmitting through. Once the photon has transmitted through, the mirror separation of the left cavity is changed to d_{low} , thus storing the photon in the optical fiber between the two cavities. When retrieving the photon, the right cavity is adjusted to have mirror separation d_{high} , which allows the photon to transmit through the cavity. This device is symmetrical, which means it also works in the same way for photons traveling from right to left. It should be emphasized that the photon is stored in the optical fiber between the two cavities and not inside either of the cavities.

In the following sections, we derive an equation for the probability of success of this device and then calculate the probability of success as a function of the total number of reflections between the two cavities, N , and the optical fiber attenuation β .

III. DERIVATION OF THE PROBABILITY OF SUCCESS

A. Deriving the probability of success in terms of reflectance, absorptance, number of reflections, fiber attenuation, and fiber length

In the classical scenario, when a laser beam with intensity I_i is incident upon a cavity, there will be a transmitted beam with intensity I_t , related via [17]

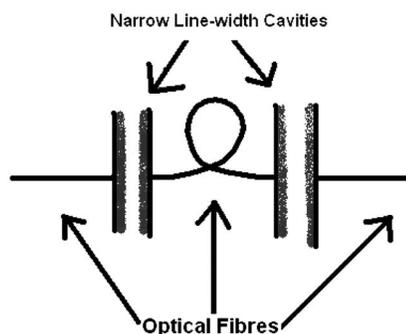


FIG. 1. A schematic diagram showing the two narrow-linewidth cavity-based quantum memory device.

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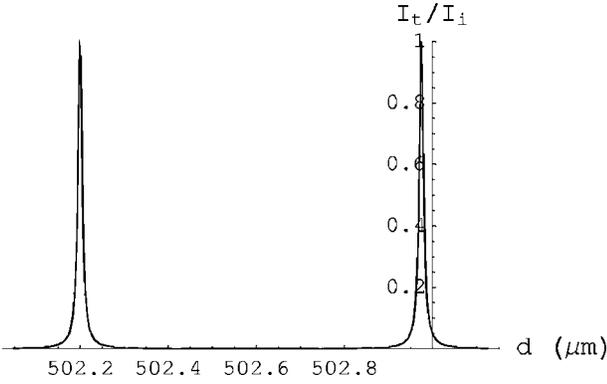


FIG. 2. The ratio of transmission intensity to incident intensity versus mirrors separation (in millimeters) plot with $R=0.9545$, $A=10^{-5}$, $n=1$, $\theta=0$, and $\lambda=1550$ nm.

$$\frac{I_t}{I_i} = \left(1 - \frac{A}{1-R}\right)^2 \left/ \left(1 + \frac{4R}{(1-R)^2} \sin^2(\delta/2)\right)\right. \quad (1)$$

The phase difference δ is related to the refractive index n of the medium inside the cavity, the mirror separation d , the transmission angle θ of the photon after passing the first mirror of the cavity, and the wavelength of the beam λ via

$$\delta = \frac{4\pi n d \cos \theta}{\lambda} \quad (2)$$

Furthermore, by the law of conservation of energy, one can show that the reflectance, transmittance, and absorptance are related via

$$R + T + A = 1. \quad (3)$$

In the quantum scenario, the ratio of the transmitted intensity to incident intensity is the probability that a photon is transmitted through the cavity, provided that the size of the photon wave packet Δx is much greater than the mirror separation d of the cavities. It will be shown later that the condition $\Delta x \gg d$ is satisfied with our scheme. Let P_{trans} be the probability that the photon is transmitted through the cavity, and correspondingly, let $P_{reflect}$ be the probability that a photon is not transmitted through but reflected back by the cavity. From this, the minimum of P_{trans} , maximum of P_{trans} , and maximum of $P_{reflect}$ can be defined as $[\frac{I_t}{I_i}]_{\delta(d_{low})}$, $[\frac{I_t}{I_i}]_{\delta(d_{high})}$, and $1 - \text{Min}(P_{trans})$, respectively. Furthermore, these three probabilities can be expressed in terms of R and A as in Eqs. (4)–(6), respectively:

$$\text{Min}(P_{trans}) = \frac{\left(1 - \frac{A}{1-R}\right)^2}{1 + \frac{4R}{(1-R)^2}} = \left(\frac{1-R-A}{1+R}\right)^2, \quad (4)$$

$$\text{Max}(P_{trans}) = \left(1 - \frac{A}{1-R}\right)^2 = \left(\frac{1-R-A}{1-R}\right)^2, \quad (5)$$

$$\text{Max}(P_{reflect}) = 1 - \left(\frac{1-R-A}{1+R}\right)^2 = \frac{(2-A)(2R+A)}{(1+R)^2}. \quad (6)$$

Another important component of this scheme is the optical fiber between the two optical cavities. Let the attenuation of this optical fiber be β (in units of dB/m) and its length be L ; then, the probability of losing the photon after a single trip between the optical cavities is $1 - 10^{-\beta L/10}$. Suppose that this device can store a photon up to N reflections between the cavities, which means the photon makes $N/2$ round trips between the cavities. Then we can define the probability of success as $\text{Max}^2(P_{trans})\text{Max}^N(P_{reflect})(10^{-\beta N L/10})$. With this definition, the probability of success is a function of R , A , N , β , and L , as described by

$$P_{success} = \left(\frac{1-R-A}{1-R}\right)^4 \left(\frac{(2-A)(2R+A)}{(1+R)^2 10^{\beta L/10}}\right)^N. \quad (7)$$

We have now derived an equation for the probability of success of our memory device. However, as will be shown soon, the necessary minimum length L of the optical fiber is dependent on the mirror separation d , the wavelength of the photon λ , and the mirror reflectance R . This means that we shall find an equation for L as a function of R and substitute it into Eq. (7) to obtain a complete equation for the probability of success.

B. Dependence of the fiber length on reflectance

The length of the optical fiber L has to be longer than the size of the photon wave packet Δx . If the time needed to adjust the mirror separation (i.e., switching between d_{high} and d_{low}) is longer than one round trip time for a photon traveling between the cavities (i.e., $2\eta\Delta x/c$), then $L = \Delta x + l$, where l is the extra length needed to buy extra time for mirror separation adjustment. However, we shall simplify our calculation by assuming that the extra length l is zero. In other words, we are assuming that one round trip time is enough for adjusting the mirror separation.

Δx is equal to $c\Delta t/\eta$, where c is the speed of light, $\eta=1.5$ is the refractive index of the fiber assumed to be independent of the wavelength bandwidth of interest, and Δt is the size of the photon wave packet in time, which we take to be approximately six standard deviations of the Gaussian photon wave packet in time.¹ Hence by the Fourier relation that the standard deviation of a Gaussian wave packet in time is the reciprocal of the standard deviation of a Gaussian wave packet in angular frequency and that a size of six standard deviations of a Gaussian wave packet in angular frequency is approximately the spectral size of the photon wave packet, we have $\Delta x = \frac{36c}{2\pi\eta\Delta f}$. Furthermore, since $c/\eta = \lambda f$, therefore $\Delta f = \frac{c\Delta\lambda}{\eta\lambda^2}$, which gives $\Delta x = \frac{18\lambda^2}{\pi\Delta\lambda}$. It will be shown in the following that $\Delta\lambda$ is related to R and thus an equation for L as a function of R can be obtained.

¹The pulse spread is found to be negligible and thus not included in Δt . With dispersion value = 18 ps per km per nm (<http://www.corning.com/photonicmaterials/pdf/pi1446.pdf>), fiber length = 200×11.65 m (i.e., 100 round trips), and wave packet uncertainty = 1.181×10^{-12} m, this gives a dispersion in time = 0.05 ps, which is negligible when compared to $\Delta t = 5.83 \times 10^{-8}$ s.

The curve of the transmission probability versus wavelength—i.e., the wavelength profile of the cavity—has to be (roughly) flat over the wavelength bandwidth of the photon wave packet to avoid distorting the shape of the photon wave packet when the photon is transmitted through or reflected by the cavity. When the mirror separation is d_{low} , with high mirror reflectance, the wavelength profile of the cavity is quite flat at the mean wavelength of the wave packet, and there will be no wave packet distortion problem. However, when the mirror separation is d_{high} , the wavelength profile of the cavity has a peak at the mean wavelength of the wave packet. To avoid the wave packet distortion problem, we need $\Delta\lambda$ to be smaller than the section of the peak where the curve is roughly flat. Here we define the width of the wavelength profile of the cavity as the horizontal width of this section of the curve at $\frac{l}{l_i} = \rho \left[\frac{l}{l_i} \right] \delta(d_{high})$ and denote it by σ . The value of ρ is set at 0.9. We give, for the expression of σ ,

$$s = \sin^{-1} \left[\sqrt{(1-\rho)(1-R)^2 / (4\rho R)} \right]$$

$$\sigma = \frac{2s(2\pi d_{high})}{(2\pi d_{high}/\lambda)^2 - s^2}. \quad (8)$$

Note that, in practice, we have $(2\pi d_{high}/\lambda)^2 \gg s^2$ and thus σ is approximately $\frac{s\lambda^2}{\pi d_{high}}$.

The value of ρ in the definition of σ is chosen arbitrarily. A larger value—say, $\rho=0.99$ —corresponds to a flatter section of the peak and leads to less distortion in the wave packet as the photons pass through the cavity. This results in a smaller σ and hence a smaller $\Delta\lambda$, because $\Delta\lambda$ has to be smaller than σ to avoid distortion. However, since $\Delta x = \frac{18\lambda^2}{\pi\Delta\lambda}$, therefore Δx will increase, which means a longer optical fiber will be needed, and by Eq. (7), the probability of success will drop accordingly. Hence the value of 0.9 is chosen as a compromise, as there is this trade-off between a higher probability of success and a lower distortion in the photon wave packet.

If we let $k = \sigma/\Delta\lambda$, then the condition that $\Delta\lambda$ has to be smaller than σ can be summarized as $k > 1$. In our calculation, we assume that $k=10$, because a larger k will cause the probability of success to drop and a lower value will induce significant distortion in the photon wave packet. With the relationship between Δx and $\Delta\lambda$, as well as the definition of σ and k , we can now express the optical fiber length L in terms of R , d_{high} , λ , ρ , k , and l , as shown in Eq. (9). Hence we now have the probability of success in terms of $R, A, N, \beta, d_{high}, \lambda, \rho, k$, and l :

$$L = \frac{9k \left[(2\pi d_{high})^2 - \left(\lambda \sin^{-1} \sqrt{\frac{(1-\rho)(1-R)^2}{4\rho R}} \right)^2 \right]}{2\pi^2 d_{high} \sin^{-1} \left(\sqrt{\frac{(1-\rho)(1-R)^2}{4\rho R}} \right)} + l. \quad (9)$$

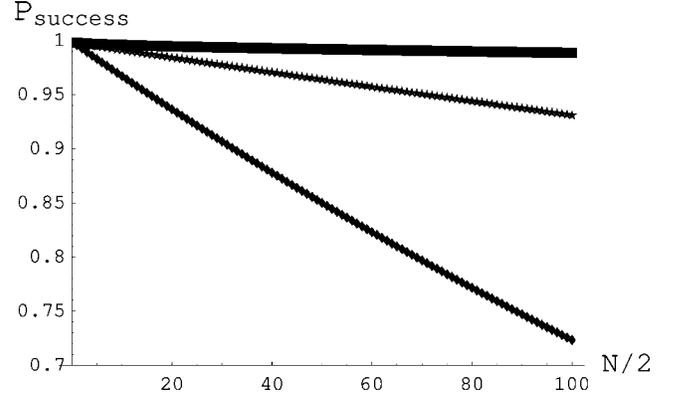


FIG. 3. A plot of the maximum probability of success vs total number of photon round trips, with parameters $A=10^{-5}$, $d=502.2 \mu\text{m}$, $\lambda=1550 \text{ nm}$, $\rho=0.9$, and $k=10$. The bottom curve has $\beta=4 \times 10^{-4} \text{ dB/m}$ [18]. The middle curve has $\beta=4 \times 10^{-5} \text{ dB/m}$. The top curve has $\beta=10^{-6} \text{ dB/m}$.

IV. RESULTS

A. Results on the probability of success against reflectance and fiber attenuation

We now have an equation for the probability of success in terms of the mirror reflection coefficient R ; the mirror absorption A ; the total number of reflections between the two cavities, N ; the optical fiber attenuation β ; the photon-transmittable mirror separation d_{high} ; the mean photon wavelength λ ; the compromised coefficient defining the width of the cavity wavelength profile, ρ ; the ratio between the width of the cavity wavelength profile and the uncertainty of the photon wavelength, k ; and the extra length of optical fiber that allows extra time for changing mirror separations, l . A state-of-the-art value for A is 10^{-5} , and λ is taken to be 1550 nm. The coefficient ρ is set as 0.9, the parameter k is chosen to be 10, and the extra length l is assumed to be zero. The parameter d_{high} is taken to be 502.2 μm because it is technically difficult to manufacture mirrors with a separation less than 0.5 mm, and the closest peak for the probability of transmission occurs at 502.2 μm . The variables N and β are treated as independent variables in our calculation. Given certain values of N and β , by plotting the probability of success versus reflectance, as in Fig. 7, a maximum value of probability of success occurs at some optimal reflectance value. Thus by varying N and β , we have obtained the four plots as shown in Figs. 3–6 for the maximum probability of success and the optimal reflectance value.

B. Result on storing a photon for 50 round trips

With the parameters $A=10^{-5}$, $d_{high}=502.2 \mu\text{m}$, $\lambda=1550 \text{ nm}$, $\rho=0.9$, $k=10$, $l=0$, and $\beta=0.0004 \text{ dB/m}$ [18], if we want to store a photon for 50 round trips between the cavities, then the maximum probability of success is 0.85 and the corresponding optimal reflectance is 0.9545 (see Fig. 3).

With the parameters $A=10^{-5}$, $R=0.9545$, $d_{high}=502.2 \mu\text{m}$, $\theta=0$, and $n=1$, we have obtained a plot of probability of success versus reflectance (see Fig. 7). This plot

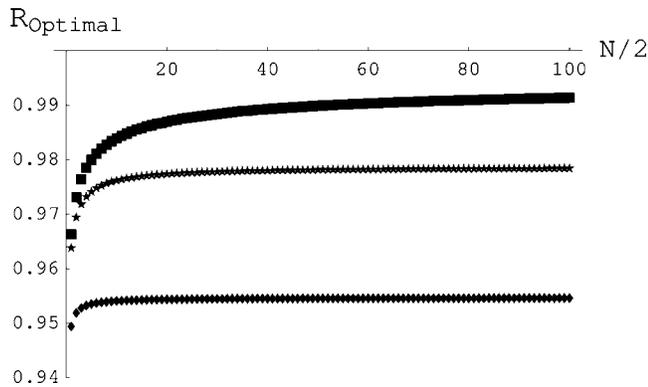


FIG. 4. A plot of optimal reflection coefficient vs total number of photon round trips, with parameters $A=10^{-5}$, $d=502.2 \mu\text{m}$, $\lambda=1550 \text{ nm}$, $\rho=0.9$, and $k=10$. The bottom curve has $\beta=4 \times 10^{-4} \text{ dB/m}$ [18]. The middle curve has $\beta=4 \times 10^{-5} \text{ dB/m}$. The top curve has $\beta=10^{-6} \text{ dB/m}$.

exemplifies the fact that there is always a maximum point on the probability of success versus reflectance graph.

Also, with the same set of parameters above, we have found $\sigma=1.181 \times 10^{-11} \text{ m}$, $\Delta\lambda=1.181 \times 10^{-12} \text{ m}$, $\Delta x=11.65 \text{ m}$, and $L=11.65 \text{ m}$, which gives a storage time of $5.886 \mu\text{s}$ for 50.5 round trips. The value of Δx is much greater than d_{high} , which confirms the fact that the light will behave as photons when passing through the mirror cavities. Using these parameters, we have also obtained a plot of the wavelength profile as shown in Fig. 8.

V. DISCUSSION

Figure 3 shows that with typical current telecom optical fibers or even with optical fibers that are 10 times smaller in attenuation, the probability of success drops rapidly as the number of photon round trips increases. The same figure also shows that an ideal optical fiber for this quantum memory scheme will have an attenuation of the order of 10^{-6} dB/m . If one wants to store a photon for 50 round trips, where the length of one round trip is about 2 times the size of the wave packet, then Fig. 5 illustrates how rapidly the maximum probability of success drops with higher attenuation in the optical fiber. Figures 4 and 6 simply tell us what mirror re-

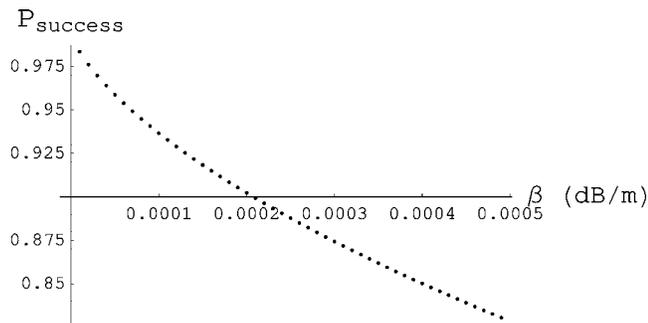


FIG. 5. A plot of the maximum probability of success vs optical fiber attenuation, with parameters $A=10^{-5}$, $N/2=50$, $d=502.2 \mu\text{m}$, $\lambda=1550 \text{ nm}$, $\rho=0.9$, and $k=10$.

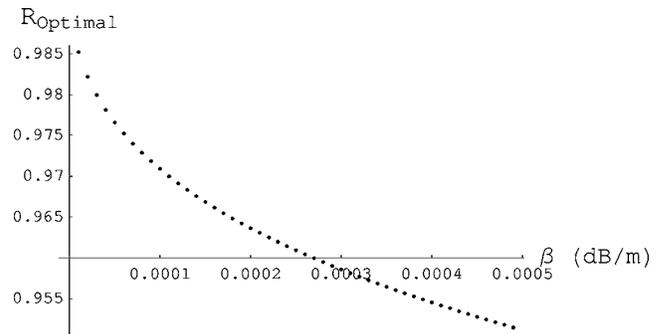


FIG. 6. A plot of optimal reflection coefficient vs optical fiber attenuation, with parameters $A=10^{-5}$, $N/2=50$, $d=502.2 \mu\text{m}$, $\lambda=1550 \text{ nm}$, $\rho=0.9$, and $k=10$.

flectance is needed to achieve the maximum probability of success, when such a quantum memory is to be built to store a photon for $N/2$ round trips in an optical fiber that has a certain attenuation.

With typical current telecom optical fiber attenuation, $\beta=0.4 \text{ dB/km}$, at $\lambda=1550 \text{ nm}$, such a quantum memory can only store a photon up to 50 round trips (about $5.8 \mu\text{s}$) with a maximum probability of success of 0.85. This may not be good enough for quantum computation purposes. There are two solutions to this problem of a low probability of success. One is to have smaller attenuation in the optical fiber and the other is to have a smaller mirror separation d_{high} . A smaller mirror separation will lead to a larger value of σ (i.e., the peak of the wavelength profile will be broader), and hence a larger wavelength uncertainty wave packet—i.e., a smaller spatial size wave packet—can be allowed. This means that a shorter optical fiber can be used and thus increases the maximum probability of success, but a short fiber will reduce the storage time. Furthermore, as mentioned earlier, such a small mirror separation is difficult to manufacture. The other alternative is to reduce the attenuation in the optical fiber and a way to reduce the attenuation is to have no optical fiber—that is, to have the photon travel in free space (and perhaps with some reflectors in between for practical reasons). This will dramatically improve the probability of success of this scheme and therefore allow many more round trips.

Another issue is to do with the wavelength profile of the cavity and the size of the wave packet. For the case of stor-

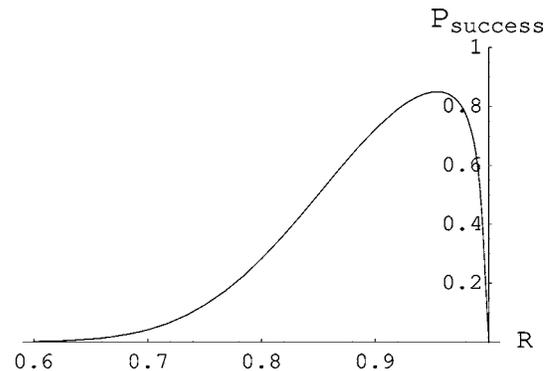


FIG. 7. Probability of success versus reflectance, plotted with parameters $A=10^{-5}$, $R=0.9545$, $d_{\text{high}}=502.2 \mu\text{m}$, $\theta=0$, and $n=1$. The maximum occurs at $R=0.9545$ and $P_{\text{success}}=0.8502$.

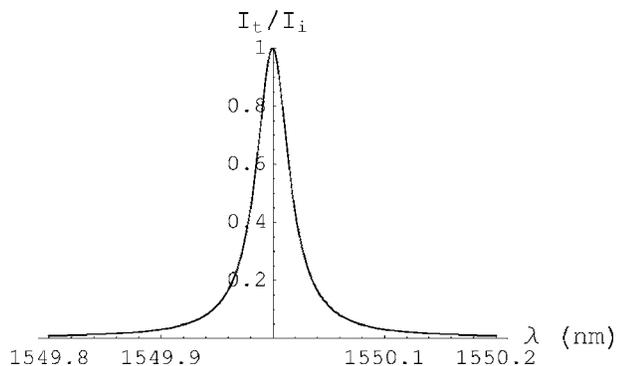


FIG. 8. The ratio of transmission intensity to incident intensity versus photon wavelength (in nm) plot, with $R=0.9545$, $A=10^{-5}$, $n=1$, $\theta=0$, and $d_{high}=502.2 \mu\text{m}$.

ing a photon for 50 round trips, the value of σ is found to be 1.181×10^{-11} m. However, with typical single-pass down-converter photons, the wavelength uncertainty is in the order of a nanometer, which is larger than σ ; hence, our scheme does not work with typical down-converter photons. However, it is possible to produce special down-converter photons with wavelength uncertainty in the order of 10^{-12} m. For instance, Shapiro and Wong [19] have produced cavity-enhanced down-converter photons, which have a wavelength uncertainty less than 10^{-14} m, and this is much better than what our scheme requires. Furthermore, photons produced by a trapped ion in a quantum electrodynamic cavity have an uncertainty in wavelength much smaller than the order of 10^{-12} m. In a recent paper by Keller *et al.* [20], the size of a photon wave packet at wavelength 866 nm, produce by a QED cavity, is about $\Delta t=1.65 \mu\text{s}$ at full width at half maximum. This corresponds to a size of $\Delta x=495$ m at full width at half maximum, which implies a wavelength uncertainty $\Delta \lambda=8.681 \times 10^{-15}$ m. Again, this is much smaller than what our scheme requires.

There is a technical problem with this scheme, which is whether it is possible to move the mirrors of the first cavity with a speed quick enough before the photon is reflected back. A solution to this problem is to have a longer optical fiber between the two cavities, such that $l \neq 0$, to act as a delay line and provide sufficient time for moving the mirrors. This means that the maximum probability of success will decrease for a fixed number of photon round trips or that the number of photon round trips will have to decrease in order to have the maximum probability of success remaining fixed.

Of course, as discussed earlier, the optical fiber may be replaced by free space, which lowers the attenuation and increases the probability of success. However, it may be difficult to implement the required length in free space.

Further considering the technical problem mentioned above, it may also be difficult to mechanically move the mirrors accurately and stably. A possible alternative to using mirrors may be to use something which is less massive and thus easier to control, such as using optical toroidal microcavities [21] to couple the photon from one fiber to another for storage. Another possible, and perhaps also better, alternative may be to use optical fibers with gratings (distributed Bragg reflectors), where the refractive index can be changed by stretching the fiber and therefore changes the transmission/reflection probability. These alternatives may be better than using mirrors in terms of accuracy and stability, as well as requiring less time for switching. The feasibility of these alternatives requires further investigation.

Another technical problem with this scheme is that there is a fiber-to-cavity coupling loss when focusing the beam with a lens. This loss occurs at two places, at one of the two sides of the two cavities. Although such a loss can be reduced by using a tailor-made aspheric lens, it is still not negligible in our scheme. A good solution is again to use fibers with gratings instead of mirror cavities, in which there is no fiber-to-cavity coupling problem.

VI. CONCLUSION

In conclusion, we estimate that our narrow-line width cavity quantum memory scheme can store a 1550-nm photon up to 50 round trips (about $5.8 \mu\text{s}$) between the cavities with a probability of success of 0.85, when the optical fiber attenuation is 0.4 dB/km. The corresponding optimal reflectance is 0.9545. The main limits on the quality of our quantum memory scheme are fiber attenuation and the speed of switching. Improved performance could be obtained with lower fiber attenuation, and a better alternative to using mirror cavities as switches may be to use microcavities or distributed Bragg reflectors.

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